Was it openness or structural reforms that rescued the Irish Economy?

Kieran McQuinn (ESRI)    Petros Varthalitis (ESRI)

2nd Workshop on Structural Reforms in EU

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Presentation overview

- Brief overview of Irish economic performance:
  - 1990 - 2008: The era of convergence (Celtic Tiger) and the emergence of imbalances
  - 2008-2014: Crisis and recovery

- Focuses on 2008-2014\(^1\) period and examines possible reasons for Ireland’s quick recovery:
  - Structural reforms
  - Trade openness and export-driven recovery

- Vehicle: A SOE-DSGE model calibrated for Ireland

\(^1\)Post 2014, well documented issues with Irish national accounts, the so called "Leprechaun economics" (see FitzGerald 2017).
Brief overview of Irish economic performance (1990-2008)

- Convergence period (1990-2000): Performance of the "Celtic Tiger" heavily depends on trade openness (see Honohan and Walsh, 2002)

- Emergence of imbalances (2000-2008):
  - Disproportionate impact of construction sector due to the credit bubble resulted in the overheating of the non-tradable sector.
  - Public finances relatively stable although buoyant tax revenues relied on the overheated non-tradable sector (e.g. construction sector).

**Construction related tax revenues**
Since 2008 Global Crisis

- Contraction in Irish output mainly due to fall in the non-tradable sector.

- Government bailed out the private sector (i.e. Irish banks) which led to a public debt surge.

- Since 2009, the main trading partners of Ireland recovered leading to an export-driven recovery of the tradable sector; while the non-tradable sector gradually recovers albeit towards a new lower trajectory.

- Between 2008-2014 few structural reforms are implemented in the Irish economy (see below OECD Indicators).
Convergence through trade openness since 90’s

Source: ESRI Macro-modelling database.
Contribution of domestic demand and net exports to real GDP growth (2001-2014)

Source: Eurostat; GDP and main components, chain linked volumes (2010).

- 2001-07 average contribution of domestic demand is 5%, and of net exports 0%.
- 2008-14 average contribution reverses to -1% for domestic demand and 2% for net exports.
Tradable and Non-tradable GVA 2001-2014

Source: Eurostat; Gross Value Added-chain linked volumes (2010), Total -all NACE activities; Notes: Gross Value Added of each sector is computed following Bergin et al. 2017.
Product and Labour Market OECD Indicators

Source: OECD; Left panel, economy-wide product market regulation index (PMR); Right panel, employment protection legislation (EPL) indicator (individual and collective dismissals-regular contracts).

Decomposition of PMR and EPL Indicators
Quantitative assessment of possible reasons for Ireland’s quick recovery using a SOE-DSGE model
Brief overview of the model

- A small open economy DSGE model.
- Three building blocks: Households, Firms and Government.
- Two sectors of production: Non-Tradable and Tradable.
  - Sector specific capital and labour factor inputs.
- Policy:
  - Monetary policy: member of a currency union,
  - Fiscal policy: independent national fiscal policy,
- The SOE faces sovereign risk premia in the world financial markets, i.e. debt elastic interest rate à la Schmitt-Grohé and Uribe (2003).
Model in equations

- Households
- Firms
  - Non-Tradable sector
  - Tradable sector
- Government
- Closing SOE
- Market Clearing Conditions
- Decentralized Equilibrium
- National fiscal policy rules
Calibration for Ireland

- We calibrate the structural parameters of the model so as the long-run solution mimics key macro variables for Ireland over the period 1995-2008.

- Fiscal data from Ireland over the period 1999-2008.
Mimicking the Irish crisis of 2008-2009 and fiscal consolidation of 2011-2013

- **Irish crisis:**
  - Exogenous adverse TFP shock in the tradable and non-tradable sector to mimic Irish GDP recession over 2008-2009 around -9%.
  - Debt shock to mimic the increase in debt-to-GDP ratio from 42% (2008) to 110% (2010).

- **Expenditure-based fiscal consolidation** (see Larch et al. 2016 European Commission Economic Briefs):
  - Spending cuts (around 4 percentage points on impact).
  - Mild temporary increases in consumption and labour effective tax rates (around 0.8% and 2% respectively).
  - Capital effective tax rate constant to its relatively low (w.r.t the rest of Eurozone) data average.
Main scenarios studied

- **Structural Reforms:**
  - Product Market Reforms (PMR) modelled as a decrease in price mark-ups of 3 and 5 pp in tradable and non-tradable sectors respectively.
  - Labour Market Reforms (LMR) modelled as a decrease in wage mark-ups of 5 pp in both sectors.
  - We experiment with the magnitude and persistence of PMR and LMR.

- **Export/RoW recovery:** Exogenous shock to Irish exports to mimic the exports’ surge observed in Irish data over 2009-10 (around 16% increase in exports-GDP ratio).

- **Baseline:** We compare the above with a "no-reform" and "no-export/RoW shock" scenario.
### Table 1: Steady state solution

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Home</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$(p^{H,F})^{1-\nu} \frac{c}{y_{GDP}}$</td>
<td>consumption as share of GDP</td>
<td>0.49</td>
<td>0.45</td>
</tr>
<tr>
<td>$(p^{H,F})^{1-\nu} \frac{d}{y_{GDP}}$</td>
<td>total public debt as share of GDP</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>$\frac{(p^{T,NT})^{1-\nu^{H}}(p^{H,F})^{1-\nu}x - rer_{c}^{F}}{(p^{H,F})^{1-\nu}y_{GDP}}$</td>
<td>Trade balance as a share of GDP</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>$\frac{(p^{T,NT})^{1-\nu^{H}}(p^{H,F})^{1-\nu}x}{rer_{c}^{F}}$</td>
<td>Exports to imports ratio</td>
<td>1.3</td>
<td>1.11</td>
</tr>
<tr>
<td>$\frac{w^{T}}{w^{NT}}$</td>
<td>Real wage ratio T w.r.t NT</td>
<td>1.5</td>
<td>1.53</td>
</tr>
</tbody>
</table>
GDP and Sectoral output

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Discussion of Key Results

- Under the export/RoW scenario GDP recession is relatively smaller on impact while GDP recovers faster.

- Implied response functions of sectoral outputs appear to move in line with post 2008 actual outcomes, i.e.:
  - Export-driven faster recovery of the tradable sector.
  - More prolonged recession and slower recovery of the non-tradable sector.

- Whereas both structural reforms scenarios can not replicate the quick GDP recovery and its sectoral breakdown, e.g. the non-tradable recovers at the same pace (or faster) with (than) the tradable sector.
Main macro variables

- Share of T
- Real Wage in T
- \( \frac{P^T}{P^NT} \)
- Trade balance
- Exports-to-GDP
The crisis seems to act as a re-balancing mechanism for the Irish economy:

- Change in GDP composition:
  - The tradable sector expands; the share of tradables (non-tradables) increases (decreases).
  - GDP recovery is driven by the tradable sector (i.e. exports).

- Any increase in overall competitiveness arises from the non-tradable sector:
  - Prices and real wages in the non-tradable sector fell because of the negative shock while prices and real wages in the tradable sector experience an export-driven increase in line with actual data.

  Sectoral real wage growth rates

- The relative price of non-tradables w.r.t tradables decreases.
Appendix A: Tables and Figures
Construction related taxation aggregates: 1985-2014

![Graph showing construction related taxation aggregates from 1985 to 2014](image)

- Stamp Duty
- CGT
- VAT

Irish economic performance (1990-2008)
Public Debt to GDP

Source: Eurostat; Government Consolidated Gross Debt (National Accounts ESA 2010).
Source: OECD product market regulation index: state control
Source: OECD product market regulation index: barriers to entrepreneurship.
Source: OECD product market regulation index: barriers to trade and investment.
Source: OECD employment protection legislation: individual dismissals.
Source: OECD employment protection legislation: collective dismissals.
Source: OECD employment protection legislation: temporary employment.
## Average growth rates of Ireland’s main trading partners

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>2008-09</th>
<th>2010-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>-1.53%</td>
<td>2.12%</td>
</tr>
<tr>
<td>UK</td>
<td>-2.33%</td>
<td>1.95%</td>
</tr>
<tr>
<td>Belgium</td>
<td>-0.73%</td>
<td>1.27%</td>
</tr>
<tr>
<td>Germany</td>
<td>-2.27%</td>
<td>2.13%</td>
</tr>
</tbody>
</table>

Source: Eurostat; Gross Domestic Product at market prices, chain linked volumes (2010).

Since 2008 Global Crisis
Average growth rates of real wages per sector

Table 3: Average real wage growth rate by sector

<table>
<thead>
<tr>
<th></th>
<th>2001-07</th>
<th>2008-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real wage per employee in the tradable sector</td>
<td>2%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Real wage per employee in the non-tradable sector</td>
<td>2.2%</td>
<td>-1.1%</td>
</tr>
</tbody>
</table>

Appendix B: Technical Appendix
The model in words

Households

- consume domestic and foreign goods
- invest in the form of physical capital in the non-tradable and tradable sector, save in the form of government bonds, lend (or borrow) in internationally traded assets and own domestic firms
- offer differentiated labour services to the non-tradable and tradable sectors; thus wages are set with a markup over households’ MRS.
Firms

- **Non-tradable sector**
  - **Final good distributors**
    - produce final non-tradable goods using a Dixit-Stiglitz aggregator
    - using as inputs intermediate non-tradable goods
    - final non-tradable goods consumed exclusively by domestic households.
  - **Intermediate good firms**
    - Enjoy market power on their own good; thus prices are set with a markup over firms’ MC
    - produce intermediate non-tradable goods using factor inputs, i.e. capital and labour
    - labour intensive sector.

- **The tradable sector is modelled similarly to the non-tradable sector.**
  - final tradable good can be consumed domestically or/and be exported to the rest-of-the-world.
  - capital intensive sector.

Brief overview of the model
Government

- imposes capital, labour and consumption taxes
- issues bonds to borrow from domestic and international financial markets
- to finance a set of spending items, i.e. public investment, purchases of goods from the private sector and transfer payments.

Brief overview of the model
Consumption bundles

Final consumption good that enters the utility function of each household $r = 1..N$ is assumed to be a composite good made of the domestic composite good, $c_t^{H,r}$, and imported goods from the RoW, $c_t^{F,r}$:

$$c_t^r = \frac{(c_t^{H,r})^\nu (c_t^{F,r})^{1-\nu}}{\nu^\nu (1 - \nu)^{1-\nu}}$$

where the domestic composite good, $c_t^{H,r}$ is made of tradables, $c_t^{T,r}$, and non-tradables goods, $c_t^{NT,r}$ produced domestically:

$$c_t^{H,r} = \frac{(c_t^{T,r})^{\nu^H} (c_t^{NT,r})^{1-\nu^H}}{(\nu^H)^{\nu^H} (1 - \nu^H)^{1-\nu^H}}$$
Price Indices

Each household $r$ allocates its total consumption expenditure, $P_t c_t^r = P_t^H c_t^{H,r} + P_t^F c_t^{F,r}$ by solving a cost minimization problem. In turn, for any level of the domestic composite good, $c_t^{H,r}$, each household $r$ allocates its total expenditure on domestic goods, $P_t^H c_t^{H,r} = P_t^T c_t^{T,r} + P_t^{NT} c_t^{NT,r}$, by solving a similar cost minimization problem. The domestic CPI is:

$$P_t = \left( P_t^H \right)^{\nu} \left( P_t^F \right)^{1-\nu}$$  \hspace{1cm} (3)

where the law of one price holds meaning that each tradable good sells the same price at home and abroad $P_t^F = S_t P_t^*$, where $P_t^*$ is the price of homogeneous imported goods produced abroad denominated in foreign currency. The domestic goods deflator is:

$$P_t^H = \left( P_t^T \right)^{\nu^H} \left( P_t^{NT} \right)^{1-\nu^H}$$  \hspace{1cm} (4)
Households

Each household $r$ chooses $\{c^r_t, l^r_t, l^{NT}_t, k^r_t, k^{NT}_t, b^r_t, f^r_t\}$ to maximize its expected discounted lifetime utility, $V^r_0$, in any given period $t$:

$$V^r_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U \left( c^r_t, l^r_t, l^{NT}_t \right)$$

subject to the sequential budget constraint in period $t$ (in nominal terms):

$$P_t (1 + \tau^c_t) c^r_t + P^T_t x^T_t + P^{NT}_t x^{NT}_t + P_t b^r_t + S_t P^* f^r_t + \Phi^* (f^r_t, f^r_{t-1})$$

$$= (1 - \tau^n_t) P_t \left( w^T_t l^T_t + w^{NT}_t l^{NT}_t \right)$$

$$+ R_{t-1} P_{t-1} b^r_{t-1} + Q_{t-1} S_{t-1} P^* f^r_{t-1} - P_t \tau^l_t$$

$$+ (1 - \tau^k_t) \left\{ P^T_t \left( r^T_t k^T_{t-1} + P_t \tilde{\omega}^{NT}_t \right) + P^{NT}_t \left( r^{NT}_t k^{NT}_{t-1} + P_t \tilde{\omega}^{NT}_t \right) \right\}$$

(6)
and to the laws of motion for physical capital:

\[
\begin{align*}
  k_t^{T,r} &= \left(1 - \delta^T\right) k_{t-1}^{T,r} + x_t^{T,r} + \Phi^T \left(k_t^{T,r}, k_{t-1}^{T,r}\right) \quad \text{(7)} \\
  k_t^{NT,r} &= \left(1 - \delta^{NT}\right) k_{t-1}^{NT,r} + x_t^{NT,r} + \Phi^{NT} \left(k_t^{NT,r}, k_{t-1}^{NT,r}\right) \quad \text{(8)}
\end{align*}
\]

where \( P_t \) is the CPI, \( l_t^{s,r}, P_t^s, x_t^{s,r}, k_t^{s,r}, r_t^{s,k}, w_t^{s,r} \) and \( \tilde{\omega}_t^{s,r} \) are hours worked, nominal price, gross investment, beginning-of-period physical capital, the real return of capital, real wage rate and real profits in sector \( s = T, NT, b, r \) and \( f_t^{*r} \) are the real value of end-of-period domestic government bonds and internationally traded assets (the latter is expressed in foreign currency) respectively, \( S_t \) is the nominal exchange rate defined as the domestic currency price of one unit of foreign currency, \( R_{t-1}, Q_{t-1} \geq 1 \) denote the gross nominal return of domestic government bonds and international assets between \( t - 1 \) and \( t \) respectively, \( \tau_t^c, \tau_t^n, \tau_t^k \) are consumption, labour and capital tax rates respectively, \( \tau_t^{l,r} \) public transfers while \( \Phi^s (.) \) introduces adjustments costs for \( s = *, T, NT \).
Production sectors

- There are two sectors of production:
- Non-tradable sector which consists of:
  - $N_{t}^{NT,h}$ final good distributors indexed by the upperscript $h = 1...N_{t}^{NT,h}$
  - $N_{t}^{NT,i}$ intermediate non-tradable good firms indexed by the upperscript $i$
- Tradable sector which consists of:
  - $N_{t}^{T,f}$ final good distributors indexed by the upperscript $f = 1...N_{t}^{T,f}$
  - $N_{t}^{T,j}$ intermediate tradable good firms indexed by the upperscript $j$
- In what follows we assume a symmetric equilibrium, i.e. $N = N_{t}^{r} = N_{t}^{NT,h} = N_{t}^{NT,i} = N_{t}^{T,f} = N_{t}^{T,j}$

Model in equations
Non-Tradable sector: A. Final good distributors

Each final good distributor maximizes its profits by choosing, \( y_{NT,i} \), while taking prices, \( P_{NT}^t \) and \( P_{NT,i}^t \), as given:

\[
P_{NT}^t y_{NT,i} = \sum_{i=0}^{N_{NT,i}} \frac{1}{N_{NT}^t} P_{NT,i}^t y_{NT,i} \tag{9}
\]

subject to a Dixit-Stiglitz type production function which combines intermediate goods of variety \( i = 1..N_{NT,i} \) into a single non-tradable good \( h \):

\[
y_{NT,h} = \lambda^{NT} \left( \sum_{h=0}^{N_{NT,i}} \left( y_{NT,i} \right)^{\varepsilon_{NT} - 1} \right)^{\frac{\varepsilon_{NT}}{\varepsilon_{NT} - 1}} \tag{10}
\]

where \( y_{NT,i} \) denotes the quantity of intermediate non-tradable good of variety \( i \), \( \varepsilon_{NT} > 0 \) is the elasticity of substitution across goods \( i \) and \( \lambda^{NT} \equiv \frac{1}{N_{NT}^t} \) is a scale parameter such that \( y_{NT,i} = y_{NT,h} \), in a symmetric equilibrium.
Non-Tradable sector: B. Intermediate good firms

Minimum cost function

Each intermediate non-tradable good firm $i$ supplies variety $i$ by solving a two-step problem. First, intermediate firm $i$ minimizes its cost by choosing its factor inputs $\{k_{t-1}^{NT,i}, I_t^{NT,i}\}$:

$$\Psi \left(y_t^{NT,i} \right) = \min \left\{ P_t^{NT} r_t k_{t-1}^{NT,i} + P_t w_t^{NT} I_t^{NT,i} \right\}$$  \hspace{1cm} (11)

taking prices as given and subject to the Dixit-Stiglitz aggregator

$$I_t^{NT,i} \equiv \frac{1}{N^r} \left( \sum_{r=0}^{N^r} \left( I_t^{NT,r} \right)^{\frac{\epsilon^{NT,w}}{\epsilon^{NT,w-1}}} \right)$$

and its technology (see below).
Non-Tradables: B. Intermediate good firms (cont’ed)

Production function

The production function is given by:

\[ y_{t,NT,i} = A_{t,NT} \{k_t^g\}^{\kappa_{NT}} \left(k_{t-1,NT,i}\right)^{a_{NT}} \times \left(l_{t,NT,i}\right)^{1-a_{NT}} \]  

(12)

Each intermediate non-tradable good firm \( i \) produces a differentiated product \( i \) utilising as inputs, \( k_t^g \), public capital, physical capital, \( k_{t-1,NT,i} \), rented from households in fully competitive markets and labour, \( l_{t,NT,i} \), services rented from households in monopolistically competitive markets. \( A_{t,NT} \), measures productivity in the non-tradable sector. Finally, \( \kappa_{NT}, a_{NT} \in [0, 1] \) are sector specific parameters.
Non-Tradables: B. Intermediate good firms (cont’ed)

Profit maximization

Firm $i$ chooses its price, $P_{NT,i}^t$, to maximize a discounted sum of expected nominal profits:

$$\max_{P_{NT}^t} \sum_{t=0}^{\infty} E_0 \mathbb{E}_{t,t+1} \left\{ P_{NT,i}^t y_{NT,i}^t - \Psi \left( y_{NT,i}^t \right) - \frac{\phi_{NT}}{2} \left( \frac{P_{NT,i}^t}{P_{NT,i}^{t-1}} - 1 \right)^2 P_{NT}^t y_{NT}^t \right\}$$

subject to demand for each variety $i$:

$$y_{NT,i}^t = \left[ \frac{P_{NT,i}^t}{P_{NT}^t} \right]^{-\epsilon_{NT}} y_{NT}^t$$

where $\mathbb{E}_{t,t+1}$ is a stochastic discount factor.
Tradable sector: A. Final good distributors

Each final good distributor maximizes its profits by choosing, $y^T_t,^f$, while taking prices, $P^T_t$ and $P^T_t,^j$, as given:

$$P^T_t y^T_t,^f = \sum_{j=0}^{N^T_t,^f} \frac{1}{N^T_t} P^{NT}_t,^j y^{NT}_t,^j$$ (15)

subject to a Dixit-Stiglitz type production function which combines intermediate goods of variety $f = 1..N^NT_t,^f$ into a single non-tradable good $^f$:

$$y^T_t,^f = \lambda^T \left( \sum_{f=0}^{N^T_t,^f} \left( y^{NT}_t,^j \frac{\varepsilon^T - 1}{\varepsilon^T} \right) \right)^{\varepsilon^T/(\varepsilon^T - 1)}$$ (16)

where $y^T_t,^j$ denotes the quantity of intermediate tradable good of variety $j$, $\varepsilon^T > 0$ is the elasticity of subsitution across goods $j$ and $\lambda^T \equiv \frac{1}{N^T_t}$ is a scale parameter such that, $y^{NT}_t,^f = y^{NT}_t,^j$, in a symmetric equilibrium.
Tradable sector: B. Intermediate good firms

Minimum cost function

Each intermediate tradable good firm $j$ supplies variety $j$ by solving a two-step problem. First, intermediate firm $j$ minimizes its cost by choosing its factor inputs $\{k_{t-1}^T, l_t^T\}$:

$$\Psi \left( y_t^{NT,j} \right) = \min \left\{ P_t^T r_t^k k_{t-1}^T + P_t w_t^T r_t^r l_t^T, j \right\}$$  \hspace{1cm} (17)

taking prices as given and subject to the Dixit-Stiglitz aggregator

$$l_t^T, j \equiv \frac{1}{N^r} \left( \sum_{r=0}^{N^r} \left( l_t^T, r \right)^{\varepsilon_{T,w-1}} \right)^{\varepsilon_{T,w}}$$

and its technology (see below).
Tradables: B. Intermediate good firms (cont’ed)

Production function

The production function is given by:

$$y_t^{T,j} = A_t^T \{ k_t^g \}^{\tau^T} \left( k_{t-1}^{T,j} \right)^{a^T} \times \left( l_t^{T,j} \right)^{1-a^T}$$  \hspace{1cm} (18)$$

Each intermediate non-tradable good firm \( i \) produces a differentiated product \( i \) utilising as inputs, \( k_t^g \), public capital, physical capital rented from households in fully competitive market, \( k_{t-1}^{T,j} \), and labour, \( l_t^{T,j} \), rented from households in monopolistically competitive market. \( A_t^T \), measures productivity in the tradable sector. Finally, \( \tau^T, a^T \in [0, 1] \) are sector specific parameters.

Model in equations
Tradable sector: B. Intermediate good firms (cont’ed)

Profit maximization

In the second step firm $j$ chooses its price, $P_{t}^{T, j}$, to maximize a discounted sum of expected nominal profits:

$$\max_{P_{t}^{T, j}} \sum_{t=0}^{\infty} E_{0} \Xi_{t, t+1} \left\{ P_{t}^{T, j} y_{t}^{T, j} - \Psi \left( y_{t}^{T, j} \right) - \frac{\phi^{T}}{2} \left( \frac{P_{t}^{T, j}}{P_{t-1}^{T, j}} - 1 \right)^{2} P_{t}^{T} y_{t}^{T} \right\}$$  (19)

subject to demand for each variety $j$:

$$y_{t}^{T, j} = \left[ \frac{P_{t}^{T, j}}{P_{t}^{NT}} \right]^{\varepsilon^{T}} y_{t}^{T}$$  (20)

where $\Xi_{t, t+1}$ is a stochastic discount factor.

Model in equations
The sequential government budget constraint in real per capita terms is:

\[ b_t + \frac{S_t P_t^*}{P_t} f_t^* g = R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + Q_{t-1} \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} \frac{P_{t-1}}{P_t} f_{t-1}^* g + g_t - \tau_t \]

(21)

where \( b_t \) is real per capita public debt held by domestic households denominated in domestic currency, \( f_t^* g \) is real per capita public debt held by foreigners denominated in foreign currency, \( g_t \) and \( \tau_t \) denote total government spending and total tax revenues in real and per capita terms respectively. In what follows, we define \( P_t b_t + S_t P_t^* f_t^* g \), where \( d_t \) denotes total public debt. Thus, \( b_t \equiv \lambda^g d_t \) and \( \frac{S_t P_t^*}{P_t} f_t^* g \equiv (1 - \lambda^g) d_t \), where \( \lambda^g \) is defined as the share of public debt held by domestic households.
**Government Spending**

Total government spending in real per capita terms is defined as:

\[ g_t \equiv g_t^g + g_t^i - \tau_t^{l,r} \] (22)

Total government spending decomposes into the three main spending items, spending on goods purchased from the private sector, \( g_t^g \), public investment, \( g_t^i \), and public transfers, \( \tau_t^{l,r} \). Notice that government purchases goods from the non-tradable and tradable sector, i.e.

\[ P_t \left( g_t^g + g_t^i \right) \equiv P_t^T g_t^T + P_t^{NT} g_t^{NT} \],

where \( g_t^T \) and \( g_t^{NT} \) denotes government purchases of tradable and non-tradable goods respectively.
Total tax revenues in real per capita terms are defined as:

\[
\tau_t = \tau^c_t c^r_t + \tau^n_t \left( \omega^T_t I^T_t,r + \omega^N_t I^N_t,r \right) \\
+ \tau^k_t \frac{P^N_t}{P_t} \left( r^k_t k^N_{t-1},r + \tilde{\omega}^N_t,r \right) + \tau^k_t \frac{P^T_t}{P_t} \left( r^k_t k^T_{t-1},r + \tilde{\omega}^T_t,r \right)
\]  (23)

Government finances its spending by collecting consumption, labour and capital taxes.
Fiscal policy instruments

The share of government consumption to GDP, $s^g_t \equiv \frac{g^g_t}{y^GDP_t}$, share of public investment to GDP, $s^i_t \equiv \frac{g^i_t}{y^GDP_t}$, the share of government purchases to total spending from the tradable sector, $\lambda^T,g_t \equiv \frac{P^T_t g^T_t}{P(t)(g^g_t + g^i_t)}$, the share of public transfers, $s^l_t \equiv \frac{P_t \tau^l,r_t}{P^H_t y^GDP_t}$, and $\tau^c_t, \tau^n_t, \tau^k_t$, are fiscal policy instruments while in each period one of the fiscal instruments needs to adjust residually to satisfy the government budget constraint (21). In what follows, we choose $d_t$ as the residual fiscal policy instrument. For the definition of $y^GDP_t$ see below.

Model in equations
Closing the Small Open Economy

To avoid non-stationarity and convergence to a well defined steady-state we endogenize the world interest rate. Following Schmitt-Grohe and Uribe (2003) we assume that the small open economy risk premium is an increasing function of the end-of-period net foreign debt as a share of nominal GDP, $\frac{S_t P^*_t (f^*_t - f^*_r)}{P_t^H y_t^GDP}$, when this share exceeds an exogenous certain threshold $\mathcal{F}$. The equation governing sovereign risk premia is:

$$Q_t = Q^*_t + \psi \left( e \frac{S_t P^*_t (f^*_t - f^*_r)}{P_t^H y_t^GDP} - \mathcal{F} - 1 \right)$$

where $Q^*_t$ denotes the world interest rate and is exogenously given, $\psi$ is a parameter which measures the elasticity of interest rate with respect to deviations of net foreign debt to GDP ratio from its threshold value.
Exports

Exports are given by the following process:

\[ x_t = \rho^x x_{t-1} + (1 - \rho^x) \bar{x} \left( \frac{P^T_t}{P^F_t} / TT \right)^{-\gamma^x} + v^x_t \]  \hspace{1cm} (25)

\( TT \) is the terms-of-trade in steady-state and is defined as the relative price of exports with respect to the price of imports. \( 0 < \rho^x < 1 \) and \( \gamma^x > 0 \) are parameters which are related to the persistence of exports and the elasticity of exports with respect to terms of trade deviations from their steady-state value.
Market Clearing conditions

Below, we solve for a symmetric equilibrium in per capita terms. Final non-tradable good:

\[
y_t^{NT} = c_t^{NT,r} + x_t^{NT,r} + g_t^{NT,g}
\]  

(26)

Final tradable good:

\[
y_t^T = c_t^{T,r} + x_t^{NT,r} + g_t^{T,g} + x_t
\]  

(27)
Market clearing conditions (cont’d)

Capital markets:

\[ k_t^{NT} \equiv k_t^{NT},r = k_t^{NT},i \]  

(28)

\[ k_t^T \equiv k_t^T,r = k_t^T,j \]  

(29)
Market clearing conditions (cont’ed)

Labour market in the non-tradable sector:

\[ l_{t}^{NT} \equiv l_{t}^{NT,i} = l_{t}^{NT,r} \] (30)

Labour market in the tradable sector:

\[ l_{t}^{T} \equiv l_{t}^{T,r} = l_{t}^{T,j} \] (31)
Market clearing conditions (cont’ed)

Government bonds market:

\[ b_t = b_t^r \] (32)
The evolution of net foreign assets (debt)

Combining the household budget constraint with the government budget constraint and substituting sequentially the clearing market conditions for tradable, non-tradable goods, labour and capital markets, the definitions of profits in the tradable and non-tradable sector yields a dynamic equation that governs the evolution of net foreign debt (assets):

\[
S_t P_t^* \left( f_t^g - f_t^r \right) = Q_{t-1} \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} \frac{P_{t-1}}{P_t} \left( f_{t-1}^g - f_{t-1}^r \right) + \frac{S_t P_t^*}{P_t} c_t^F r - \frac{P_t^F}{P_t} x_t + adj_t
\]

(33)

where \( adj_t \) is the sum of the adjustment costs imposed above.
Domestic GDP

We define domestic GDP as the sum of value added in the tradable and non-tradable sector:

\[ p_t^H y_t^{GDP} \equiv p_t^T y_t^T + p_t^{NT} y_t^{NT} \]  \hspace{1cm} (34)
The Decentralized Equilibrium (DE) is defined as a sequence of 39 endogenous variables \( \{ c^r_t, l^r_t, k^{T,r}_t, k^{NT,r}_t, f^*_r, c^H_r, c^{F,r}_t, c^{NT,r}_t, c^{T,r}_t, \Lambda_t, y^{NT}_t, l^{NT}_t, \tilde{\omega}^{NT}_t, \Psi^{NT}_t, y^T_t, l^T_t, \tilde{\omega}^T_t, r^{k,T}_t, \Psi'_T, k^g_t, P_t, P^{NT}_t, P^T_t, P^F_t, P^H_t, w^T_t, w^{NT}_t, r^{k,NT}_t, Q_t, x^{T,r}_t, x^{NT,r}_t, d_t, R_t, g_t, g^T_t, g^{NT}_t, \tau_t, \lambda^T_t, \tau^l_t, g^∞_t \}_t=0 \) in 39 equations given the policy instruments \( \{ \tau^c_t, \tau^n_t, \tau^k_t, s^g_t, s^i_t, \lambda^T_t, g^∞_t, \tau^l_t \}_t=0 \) the exchange rate regime\(^2\), the rest-of-the-world variables \( \{ P^*_t \}_t=0 \), the exogenous shocks \( \{ A^T_t, A^{NT}_t \}_t=0 \) and initial values for the state variables.

\(^2\)In a currency union \( \{ R_t \}_t=0 \) is endogenous and \( \{ S_t \}_t=0 \) exogenous.
National fiscal policy rules

\[ s_g^t - s^g = -\gamma_l^g \left( \frac{P_{t-1}d_{t-1}}{P_{t-1}^H y_{t-1}} - \bar{l} \right) - \gamma_y^g \left( y_t^{GDP} - \bar{y}^{GDP} \right) \]  

\[ s_i^t - s^i = -\gamma_l^i \left( \frac{P_{t-1}d_{t-1}}{P_{t-1}^H y_{t-1}} - \bar{l} \right) - \gamma_y^i \left( y_t^{GDP} - \bar{y}^{GDP} \right) \]  

\[ \tau_c^t - \tau_c = \gamma_l^c \left( \frac{P_{t-1}d_{t-1}}{P_{t-1}^H y_{t-1}} - \bar{l} \right) + \gamma_y^c \left( y_t^{GDP} - \bar{y}^{GDP} \right) \]  

\[ \tau_k^t - \tau_k = \gamma_l^k \left( \frac{P_{t-1}d_{t-1}}{P_{t-1}^H y_{t-1}} - \bar{l} \right) + \gamma_y^k \left( y_t^{GDP} - \bar{y}^{GDP} \right) \]  

\[ \tau_n^t - \tau_n = \gamma_l^n \left( \frac{P_{t-1}d_{t-1}}{P_{t-1}^H y_{t-1}} - \bar{l} \right) + \gamma_y^n \left( y_t^{GDP} - \bar{y}^{GDP} \right) \]  

where \( \gamma_{l,y}^q \geq 0 \) for \( q \equiv \{ g, i, c, k, n \} \) are feedback policy coefficients and \( \bar{l} \) and \( \bar{y}^{GDP} \) are debt-to-GDP and output targets respectively.